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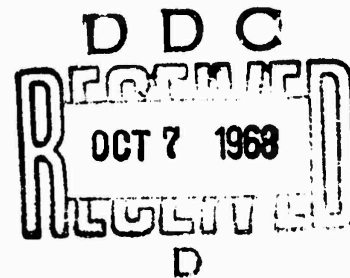
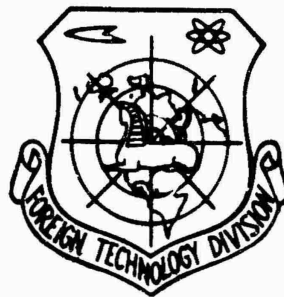
FOREIGN TECHNOLOGY DIVISION



THE ORBITS OF INTERPLANETARY NAVIGATION (CHAPTER 6)

by

Hsueh-Sen Ch'ien



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EDITED TRANSLATION

THE ORBITS OF INTERPLANETARY NAVIGATION (CHAPTER 6)

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ABSTRACT: The distance from the center of the earth at which the gravitational forces due to the sun and earth are equal is calculated from the basic equation

as $d = 2.6 \times 10^5$ km. The optimal (Hohmann) orbit of sending a spacecraft from the orbit of the earth to the orbit of a planet is considered. This type of orbit and the required rocket power are discussed. The total incremental velocity required of sending a spacecraft from the orbit of the earth to the orbit of a planet is derived as

$$\Delta v = v_2^* \left[\sqrt{\frac{2r_1}{r_1 + r_2}} - 1 \right] + v_1^* \left[1 - \sqrt{\frac{2r_2}{r_1 + r_2}} \right]$$

The path traveled by the spacecraft is one half of an elliptical orbit. Hence,

$$T^* = \frac{1}{2} T = 2\pi \sqrt{\frac{R}{g}} \cdot \frac{1}{2} \cdot \left(\frac{r_1 + r_2}{2R} \right)^{3/2} \sqrt{\frac{M}{M_\odot}} = \frac{84.5}{2} \left(\frac{r_1 + r_2}{2R} \right)^{3/2} \sqrt{\frac{M}{M_\odot}} (\text{sec})$$

Where v_2^* : Orbital velocity of the earth.

v_1^* : Orbital velocity of the planet.

r_1 : Distance from the sun to perigee.

r_2 : Distance from the sun to apogee.

M_\odot : Mass of the sun.

M : Mass of the earth.

g_\odot : Gravitational constant of the sun.

g : Gravitational constant of the earth.

R_\odot : Radius of the sun.

R : Radius of the earth.

A numerical example of launching a spacecraft from the earth to Jupiter either

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directly from the earth or from an artificial satellite is given. The advantage of using low thrust rocket engine to launch the spacecraft from a satellite based on two fundamental equations is discussed, and followed by a mathematical analysis. The equations used are

$$\frac{d^2 r}{dt^2} - r \left(\frac{d\theta}{dt} \right)^2 - g^* \left(\frac{r^*}{r} \right)^2 + F_r = 0$$

$$\frac{d}{dt} \left(r^2 \frac{d\theta}{dt} \right) = r F_\theta$$

where F_r is the instantaneous radial thrust,

F_θ is the instantaneous tangential thrust,

r^* is the radius of satellite orbit,

r is the distance from the center of the earth to the spacecraft in free path,

and g^* is the gravitational constant in the orbit of a satellite. A real time calculation of the acceleration stage is given by using previous analysis. Also, a spiral orbit during the acceleration is described. Utilization of optical pressure to produce very little acceleration for an interplanetary spacecraft is suggested. A brief calculation of optical pressure near the earth is made. Orig. art. has: 38 formulas and 5 figures. English translation: 27 pages.

Chapter 6

THE ORBITS OF INTERPLANETARY NAVIGATION

§6.1. THE GRAVITATIONAL FIELD OF THE SUN

We have discussed in detail the motion of a particle in the central force field, and also described the motion of a rocket under the action of the gravitational field of the earth. Here, we shall study the flight of a spaceship in the gravitational field of the sun after it has left the gravitational force of the earth. The motion under the solar gravitational field is, in basic principle, similar to what is in the earth's gravitational field. Therefore, in discussing the motion under the action of the solar gravitational field, it is possible to use the basic principles presented in the last chapter on the motion in the central force field in its entirety.

In Chapter 5, we found the centripetal force constant of the earth to be $\mu^2 = gR^2$. In considering the motion of a body around the sun, we need to apply a correction to this value. From the law of universal gravitation, we know that when a body located at a distance of r from the sun moves around the sun, the centripetal force received by its unit mass is $g_0 \left(\frac{R_0}{r} \right)^2$; g_0 is the gravitational acceleration on the surface of the sun, R_0 is the radius of the sun. Here it is easy to derive $g_0 = \frac{R^2}{M} \frac{M_0}{R_0^2}$; where M and R are, respectively, the mass and radius of the earth. Hence, we obtain

$$g_0 \left(\frac{R_0}{r} \right)^2 = \frac{R^2}{M} \frac{M_0}{R_0^2} \left(\frac{R_0}{r} \right)^2 = \frac{M_0}{M} \left(\frac{R}{r} \right)^2 = \frac{\mu_0^2}{r^2},$$

Therefore, the centripetal force constant of the sun is

$$\mu_0 = \epsilon \frac{M_0}{M} R^2. \quad (6.1)$$

Having done this, we can now apply the basic equations in Chapter 5 to calculate the orbits of the spaceship in the solar gravitational field.

Here, we can calculate first, at what distance from the center of the earth, will the gravitational field of the sun be equivalent to the earth's gravitational field? Since the distance from the earth to the sun is approximately 1.495×10^8 kilometers; $M_0/M = 332,488$. If this distance is d then

$$\epsilon \frac{R^2}{d^2} = \epsilon \frac{M_0}{M} \frac{R^2}{(1.495 \times 10^8)^2},$$

that is

$$d = 1.495 \times 10^8 \sqrt{\frac{1}{332,488}} = 260,000 \text{ kilometers}$$

if we take into account also the action of the moon, then, once a spaceship is more than four hundred thousand kilometers away from the earth, it has basically left the gravitational pull of the earth, the moon and its motion is principally controlled by the solar gravitational field.

§6.2. THE ELLIPTICAL ORBIT IN THE SOLAR SYSTEM

The orbits we will be discussing here are the most economical orbits to send a spaceship from the earth orbit to the orbits of other planets, the so-called double tangent orbit (supposedly first proposed by Walter Hohmann). In this section, we shall describe these orbits and the motive force required for the rockets.

It is common knowledge that the orbits of the various planets around the sun are all nearly circular elliptical orbits having the sun as their focal point. Also, their orbital planes are basically on the same plane. Hence, we may consider these orbits to be circular and

all in one common plane. Thus, the problem is very much simplified. Figure 6.1 represents the navigation route of the spaceship from the earth's orbit to those of the outer layer planets. In the above chapter, we have already established that the orbit that is the most economical in motive power from a point at a definite height from the earth's surface to another relatively higher circular orbit of a satellite, is an elliptical orbit having the perigee as the definite height from the earth's surface and the apogee as the height of the orbit to be reached by the satellite. This result may be extended to the problem of interplanetary orbits, except the perigee ought to be perihelion and the apogee should be aphelion, or, that is to say, the best orbit to reach the planets further than the distance from the earth to the sun should be the elliptical orbit circumscribing the earth's orbit, inscribed to the orbit of the planet, and having the sun as its focal point. This is the so-called interplanetary navigation orbit. The perihelion of this orbit is exactly on the earth's orbit while its aphelion is on the orbit of the planet.

After having left the gravitational field of the earth, although the spaceship no longer has any relative velocity to the earth, it is, nevertheless, possible that it could move around the sun with the earth on the earth's orbit. In order to make it leave the earth's orbit and move along the elliptical orbit shown in Fig. 6.1, the energy possessed by the spaceship originally is certainly not sufficient. It is necessary to impart to it additional velocity or to make it so that at the moment it leaves the earth's gravitational field, the remaining segment of velocity is enough for it to coast into a free flight on the elliptical orbit, and reach the aphelion of the elliptical orbit; that is to reach the point of tangency between the elliptical orbit and the orbit of the planet in the outer circle. At this moment, it is again necessary

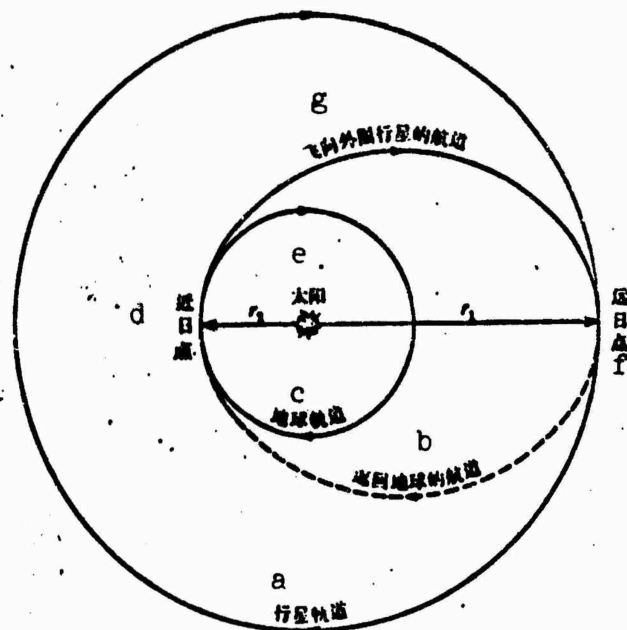


Fig. 6.1. The spaceship on the navigational route between the earth's orbit and the orbit of the planet. a) Orbit of the planet; b) re-entry orbit; c) earth's orbit; d) perihelion; e) sun; f) aphelion; g) flight orbit towards the planet.

to impart additional energy to the spaceship, so that it will have the necessary orbit velocity to enter the orbit of the planet. Supposing we do not give it that extra amount of energy, then it will not be able to enter the orbit of the planet owing to insufficient energy and will move along the length of the other arc of the elliptical orbit toward the perihelion, then it will be going along this elliptical orbit and become an artificial satellite of the sun. Here we can see that in order to send a cosmic ship from the earth's surface to the other planets, it is necessary to divide the orbit into two stages of acceleration with a segment of free flight in between to achieve this purpose. Of course, if we want to shorten the flight time, it is also possible to use other orbits. But these orbits would require still higher velocity for the spaceship. The level reached by present-day rocket technology permits only the use of the most economical orbit, namely the use of the double tangent orbit or orbits similar to it. In 1961, the orbit used by the

Soviet Union in their rocket launched to Venus was one that required less time than the one of the least motive power, that is, it took more power but less flight time.

Here we have only explained the most practical and most economical orbit used in launching an interplanetary spaceship to the planets outside the earth, but what kind of an orbit should we use in launching it to the planets in the inner circle closer to the sun: Venus and Mercury? This problem is rather simple. It is exactly similar to the elliptical orbit used for launching a spaceship to the planets in the outer circle. But, the launching direction is exactly the opposite. That is to say, from the earth's orbit, the spaceship must be made to enter into an elliptical orbit which is tangent to the earth's orbit at the aphelion and tangent to the orbit of Venus or Mercury at the perihelion. This then requires a deceleration of the spaceship so that it can be made to enter the elliptical orbit, then after the spaceship has reached the point of tangency with the orbit of the planet, another deceleration is applied so that it will lose more energy and enter the orbit of the planet.

In the above, we have described the orbits of interplanetary spaceships launched from the earth's orbit to other planets in detail. Now, we shall proceed into calculations. In Chapter 5, we had an elliptic equation

$$r = \frac{p}{1 + e \cos(\theta - \theta_0)}, \quad (6.2)$$

Let r_1 be the distance of the aphelion on the ellipse and r_2 be the distance of the perihelion on it (see Fig. 6.1). We obtain

$$\frac{r_2}{r_1} = \frac{1 - e}{1 + e}.$$

Solving it we get the eccentricity of the ellipse

$$e = \frac{r_1 - r_2}{r_1 + r_2};$$

therefore,

$$\sqrt{1 - e^2} = \frac{2\sqrt{r_1 r_2}}{r_1 + r_2}.$$

If a is the major axis of the ellipse and b the minor axis then,

$$a = \frac{r_1 + r_2}{2}, \quad b = a\sqrt{1 - e^2};$$

Therefore the area of the ellipse is

$$\pi ab = \pi a^2 \sqrt{1 - e^2} = \pi \left(\frac{r_1 + r_2}{2} \right) \sqrt{r_1 r_2}. \quad (6.3)$$

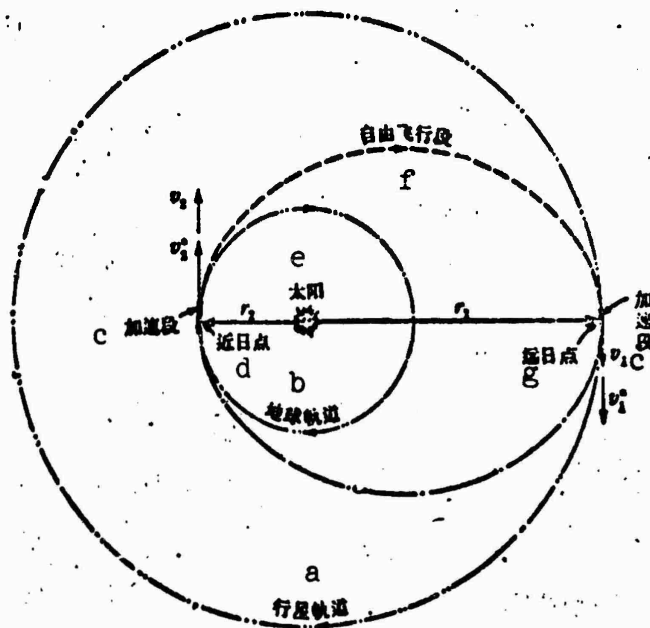


Fig. 6.2. Hohmann type interplanetary navigation route. a) Orbit of the planet; b) earth's orbit; c) acceleration stage; d) perihelion; e) sun; f) free flying stage; g) aphelion.

From Eq. (5.15) we obtained the period of the motion of a particle around the ellipse to be

$$T = 2\pi \frac{a^{3/2}}{\mu_0} = 2\pi \left(\frac{r_1 + r_2}{2} \right)^{3/2} \frac{1}{\mu_0}. \quad (6.4)$$

From Eqs. (6.3) and (6.4), according to the principle that for the particle, in unit time, the area swept by its radius does not vary and the period of its motion also does not vary, we get

$$\text{area velocity} = \frac{\text{area of ellipse}}{\text{period}} = \frac{\mu_0}{2} \frac{\sqrt{r_1 r_2}}{\sqrt{\frac{r_1 + r_2}{2}}} = \text{constant} \quad (6.5)$$

Thereby, we will be able to find the velocity of the spaceship at the perihelion and aphelion when it is flying along the elliptical orbit.

Supposing v_2^* is the orbit velocity of the earth; v_2 is the orbit velocity of the spaceship at the perihelion when it is moving along the elliptical orbit; v_1^* the orbit velocity of the planet; v_1 the velocity of the spaceship when it reaches the aphelion while flying along the elliptical orbit, (see Fig. 6.2). Hence from Eq. (6.5) we can obtain

$$\text{area velocity} = \frac{1}{2} r_1 v_1 = \frac{1}{2} r_2 v_2 = \frac{\mu_0}{2} \frac{\sqrt{r_1 r_2}}{\sqrt{\frac{r_1 + r_2}{2}}};$$

therefore the velocity of the spaceship at the perihelion and the aphelion, while flying along the elliptical orbit, may be found

$$v_1 = \mu_0 \left(\frac{r_2}{r_1} \right)^{1/2} \frac{1}{\sqrt{\frac{r_1 + r_2}{2}}}; \quad (6.6)$$

$$v_2 = \mu_0 \left(\frac{r_1}{r_2} \right)^{1/2} \frac{1}{\sqrt{\frac{r_1 + r_2}{2}}}. \quad (6.7)$$

In the above, we have already explained the reason for the requirement of higher velocity for the spaceship to enter the elliptical orbit, let this velocity increment be Δv_1 , then

$$\begin{aligned} \Delta v_1 &= v_1 - v_1^* = \mu_0 \left(\frac{r_1}{r_2} \right)^{1/2} \frac{1}{\sqrt{\frac{r_1 + r_2}{2}}} - \frac{\mu_0}{\sqrt{r_2}} \\ &= \frac{\mu_0}{\sqrt{r_2}} \left[\sqrt{\frac{2r_1}{r_1 + r_2}} - 1 \right] = v_1^* \left[\sqrt{\frac{2r_1}{r_1 + r_2}} - 1 \right]; \end{aligned} \quad (6.8)$$

Similarly, let Δv_1 be the velocity increment required for the spaceship to enter into the orbit of the planet from its elliptical orbit

$$\Delta v_1 = v_1^* - v_1 = \frac{\mu_0}{\sqrt{r_1}} \left[1 - \sqrt{\frac{2r_2}{r_1 + r_2}} \right] - v_1^* \left[1 - \sqrt{\frac{2r_2}{r_1 + r_2}} \right]. \quad (6.9)$$

From Eq. (6.8), one can see, $2r_1 > r_1 + r_2$; therefore, $\left[\sqrt{\frac{2r_1}{r_1 + r_2}} - 1 \right] > 0$, which explains that Δv_2 is positive, i.e., in order to make the spaceship in the earth's orbit to enter the elliptical orbit for the launch into the planets in the outer circle, it is necessary to increase its velocity. Similarly, from (6.9), one can see $2r_2 < r_1 + r_2$, therefore, $\left[1 - \sqrt{\frac{2r_2}{r_1 + r_2}} \right] > 0$, explaining that Δv_1 is also positive, i.e., when the spaceship enters the orbit of the planet in the outer circle from its elliptical orbit, increased velocity is also necessary. Consequently, it is very easy for us to derive that the condition for the launching of a spaceship from the earth's orbit into the orbit of Venus or Mercury is exactly the reverse, namely, a decrease in velocity is necessary for it to go from the earth's orbit into the elliptical orbit and another decrease of velocity for the entry into the orbit of the planet in the inner circle from the elliptical orbit.

Hence, we can proceed to calculate the required total increase in velocity for the interplanetary spaceship to be launched from the earth's orbit into the orbit of the other planets

$$\Delta V = \frac{\mu_0}{\sqrt{r_1}} \left[\sqrt{\frac{2r_1}{r_1 + r_2}} - 1 \right] + \frac{\mu_0}{\sqrt{r_1}} \left[1 - \sqrt{\frac{2r_2}{r_1 + r_2}} \right],$$

It may be written as

$$\Delta V = v_1^* \left[\sqrt{\frac{2r_1}{r_1 + r_2}} - 1 \right] + v_1^* \left[1 - \sqrt{\frac{2r_2}{r_1 + r_2}} \right]. \quad (6.10)$$

From the earth's orbit going to the orbit of the other planets, the path from the start to the finish is exactly half of the elliptical orbit, therefore, we find the flying period of the spaceship is a half-

period, namely

$$\begin{aligned}
 T^* &= \frac{1}{2} T = \pi \left(\frac{r_1 + r_2}{2} \right)^{3/2} \frac{1}{\mu_0} = \pi \left(\frac{r_1 + r_2}{2} \right)^{3/2} \frac{1}{R \sqrt{\frac{M_0}{M}}} \\
 &= \pi \left(\frac{r_1 + r_2}{2R} \right)^{3/2} \sqrt{\frac{R}{M_0}} = 2\pi \sqrt{\frac{R}{M_0}} \cdot \frac{1}{2} \cdot \left(\frac{r_1 + r_2}{2R} \right)^{3/2} \sqrt{\frac{M}{M_0}}, \\
 T^* &= \frac{84.5}{2} \left(\frac{r_1 + r_2}{2R} \right)^{3/2} \sqrt{\frac{M}{M_0}} \text{ (分)}. \quad (6.11)
 \end{aligned}$$

In the derivation of the above formulas, although they are based on the model of launching a spaceship from the earth's orbit into the orbits of the planets in the outer circle they are, however, applicable to calculations for the elliptical orbits between the orbits of any two arbitrary planets

§6.3. ACTUAL EXAMPLES

We shall use, as an example, the launching of an interplanetary spaceship from the earth's orbit to Jupiter to explain some of the pertinent concrete problems.

From Table 1.5, we can get the following values: $V_1 = \sqrt{gR} = 7.91$ km/sec; $R = 6371$ km; $r_2 = 1.49457 \times 10^8$ km; $r_1 = 5203r_2$; $M_0/M = 332,488$; therefore, from the above equation we find by calculation

$$v_2^* = 7.91 \times 3.764 = 29.80 \text{ km/sec}; v_1^* = 13.06 \text{ km/sec}$$

$$\Delta v_1 = v_1^* \left[\sqrt{\frac{10.406}{6.203}} - 1 \right] = 8.82 \text{ km/sec}; \Delta v_2 = 5.64 \text{ km/sec}$$

Here we have only computed the velocity increase necessary for the spaceship while it is flying along the elliptical orbit. But, we know that spaceships flying towards other planets will also have to start the flight from the surface of the earth, hence we have to take the process of the launching from the earth's surface and consider it together with the flight along the double tangent path. In the following, we shall calculate two different schemes of launching a spaceship to Jupi-

The first scheme: the spaceship starts the flight directly from the earth's surface, that is, giving the spaceship a velocity enabling it to leave the earth's gravitational field at a relatively small distance from the earth and still retaining a definite velocity enabling it to enter directly into the elliptical orbit flying straight to the orbit of Jupiter. When it reaches the position where the elliptical orbit is tangent to the Jupiter orbit, it is given an increase in velocity enabling it to enter the Jupiter orbit. Actually, it is dividing the total path of flying from the earth's surface to Jupiter into two stages of acceleration with an intermediate stage of free flight. Consequently, we can calculate the energy required by the spaceship for the first stage of acceleration

$$\frac{1}{2} (\Delta v)^2 = \frac{1}{2} v_i^2 + \frac{1}{2} (\Delta v_1)^2;$$

therefore, the velocity increment required for the first stage of acceleration is

$$\Delta v = \sqrt{v_i^2 + (\Delta v_1)^2} = \sqrt{11.18^2 + 8.82^2} = 14.23 \text{ km/sec}$$

whereupon the increase in velocity necessary for the entire flight is

$$V = \Delta v + \Delta v_1 = 14.23 + 5.64 = 19.87 \text{ km/sec}$$

The second scheme: an artificial satellite is launched first from the earth's surface. Then, from this artificial satellite the spaceship is launched along a similar elliptical orbit towards Jupiter; namely, the total flight is divided into two major stages. Firstly, the method of launching satellites as discussed in Chapter 5 is used to launch a satellite, then, the method of the double tangent orbit is used to launch a spaceship from the satellite to Jupiter. Consequently, we can calculate the total energy that the spaceship should have for the entire flight

$$\begin{aligned} v &= [v_i^2 + \{\sqrt{v_i^2 - v_1^2 + (\Delta v_1)^2}\}^2]^{1/2} + \Delta v_1 \\ &= + \sqrt{11.18^2 + 8.82^2} + 5.64 = 14.23 \text{ km/sec.} \end{aligned}$$

From the results of these two schemes one can clearly see: the energy demand for the two schemes is the same. There are, however, great differences in the method of execution; the first scheme is relatively simpler. There is the possibility of using just one continuous stage of velocity increase while the second scheme calls for the use of three stages of velocity increase in mutually separated intervals. We know that the Venus rocket launched by the Soviet Union was launched from a satellite. Since this method is so complicated, why is it then that the direct launching from the earth was not adopted? The reason is that although the second scheme requires a multistage acceleration, but the lifetime of the artificial satellite could be fairly long hence its orbit parameters could be very accurately determined, and also there will be time to take advantage of these parameters to calculate on the ground, the exact moment to fire the interplanetary rocket, the direction of the action of the thrust and the execution of this launching. In all, the launching of a cosmic ship from an artificial satellite can be done with the highest accuracy and thereby increase greatly the reliability and precision of the flight orbit. On the other hand, although the power demand in the first method is relatively low, but since there is no such intermediary as the artificial satellite to proceed with the accurate survey of the orbit and aiming, control is very difficult. Launching is not reliable. Certainly, after the solution of the problem of the precision control of still more multistage rockets, the method of directly launching can also be turned into reality.

The flight time of the spaceship to Jupiter can be calculated from Formula (6.11)

$$T^* = \frac{84.5}{2} \left(\frac{6.203 \times 1.49457 \times 10^8}{12,742} \right)^{1/2} \sqrt{\frac{1}{332.488}}$$

$$= 1,438,000 \text{ min} = 1,000 \text{ days}$$

Here the portion of the launching from the earth's surface constitutes but a very small segment of the entire flight, therefore, in the calculation of the flight time, only the time required for the flight along the path of the double tangent orbit needs to be considered.

From the above calculation, it is seen that the use of the double tangent orbit in launching interplanetary spaceships consumes far less energy in comparison with other orbits. But, from the standpoint of time, it is fairly long, flying from the earth to Jupiter takes three years. If we can improve more effectively the thrust system, then the rocket can reach a greater height such that other types of orbits may be used, e.g., the parabolic orbit and the hyperbolic orbit. But, we know that when the rocket is flying towards the planet, it is still affected by the orbital velocity of the earth. If the velocity of the spaceship is too small, it will make straight line or close to straight line flights impossible. Consequently, if it is desired to shorten the time of the flight, the rockets should have extremely high velocity, e.g., 50 km/sec. Under the conditions attainable by present-day rocket engines, the velocity that may be used for interplanetary navigation is but 20 km/sec and less, that is using the double tangent orbit or orbits approximating this situation.

Finally, it also needs to mention an energy saving device of launching a solar explorer (an automatic station close to the sun). It has been proposed that one may launch the rocket at a velocity slightly lower than the third cosmic velocity, so that the rocket will finally reach the edge of the solar gravitational field and revolve around the sun. Since at this moment its orbital velocity around the sun becomes very small, it is only necessary to compensate that velocity and it will automatically fall toward the center of the sun due to its gravitational pull. But, although the energy required is quite low, it does

require very accurate control since in compensating the velocity it has to be done exactly right. Overcompensation will make it revolve around the sun in a reverse direction and undercompensation will result in its continuing the revolution around the sun, and unable to fall straight toward the sun. Hence, to have to aim at the planet where it is supposed to be going at this immense distance makes it a still more difficult task. To try to realize this proposal is a most difficult and complex procedure. It will have to wait for still further technical development to resolve this problem.

§6.4. LOW THRUST ORBIT IN THE CENTRAL FORCE FIELD

Low thrust orbits are based on this type of reasoning: owing to the consideration of the numerous advantages of launching off a satellite orbit, for example, what we have cited earlier: starting the flight from the satellite in comparison with the flying off the surface of the earth, affords a higher degree of precision in control, therefore, many interplanetary navigation orbits start from the orbits of the earth satellites. In the satellite orbit, the gravitational force of the earth with respect to the satellite always exists since the satellite is revolving around the earth, it must have acceleration and this acceleration is exactly equal to that produced by the gravitational force of the earth. At the same time, the objects on the artificial satellite are also revolving around the earth, their accelerations are also exactly the same as that produced by the earth's gravitational force. Hence, there will not be the need of any force to uphold the objects inside the satellite. The objects are in a weightless state. While they are weightless they are nevertheless still under the action of the gravitational force of the earth. Here we may see that although the weight of an object originates from the earth's gravitation, but it is not the same thing as the gravitational force. We need to have a clear un-

derstanding of this concept. Consequently, there are people who think that it may be possible to make use of the fact of weightlessness to employ a very small acceleration to launch a spaceship from the orbit of a satellite; they have also proposed that in interplanetary navigation, it is possible to use low thrust orbits, employing an electric rocket system of low thrust but extremely high pulse ratio (we shall discuss this in the next chapter). This presents problems in a different area, namely the investigation of orbits with acceleration lower than $1/1000$ g to $1/10,000$ g. But, in order to use acceleration as low as that, it can only be done at the expense of something. Also with further lowering of the acceleration, the cost gets higher and higher which will be clear in the calculations later. Therefore, we cannot assume that on the satellite orbit where there is weightlessness, there will not be any more weight expenditure. This is incorrect. Actually, although on the satellite orbit, the sense of the gravitational force is lost but the satellite has not left the earth's gravitational field and just as it was mentioned in §5.2, there is still expenditure of weight, consequently, low thrust orbits still have to pay a definite price. Below we shall carry out a concrete calculation to explain the launching orbit of the satellite, or the low acceleration orbits. Since the thrust is consistently acting on the flying vehicle therefore, the flying orbit is no longer an ellipse but has become a spiral orbit.

Assume here g^* is the gravitational constant on the satellite orbit; F_r is the radial thrust of the mass of the spaceship in unit instant; F_θ is the circular thrust of the mass of the spaceship in unit instant (see Fig. 6.3). The values F_r , F_θ can vary during the flight. Applying Newton's law we can write the following two equations of motion on flying vehicle of unit mass to serve as the basis of the problem we are considering. The radial velocity of the flying vehicle is

$$\frac{d^2 r}{dt^2} = r \left(\frac{d\theta}{dt} \right)^2 - g^* \left(\frac{r^*}{r} \right)^3 + F_r. \quad (6.12)$$

From the variation of the angular momentum in unit time, we can obtain the equation for the circular motion as

$$\frac{d}{dt} \left(r^2 \frac{d\theta}{dt} \right) = r F_\theta. \quad (6.13)$$

According to the results of the previous calculations, we know that using the radial thrust direction leads to a relatively larger consumption of energy and the effect is not good. The best thrust direction is also not the direction of the tangent to the orbit but is a direction where it forms a desirable angle with the tangential direction of the orbit. But the difference in the mass ratio required in the circular direction and the direction of the optimum thrust is not very great, therefore, we are considering here only the circular thrust to simplify the problem. The results obtained are also very close to those obtained in the optimum thrust direction. For the convenience of calculation, we have adopted here the following format of dimensionless variables.*

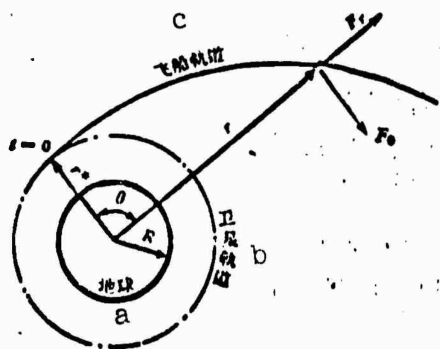


Fig. 6.3. Launching from the satellite orbit. a) Earth; b) spaceship orbit; c) satellite orbit.

The dimensionless quantity for length is

$$\rho = \frac{r}{r^*}. \quad (6.14)$$

here r is the distance from the flying orbit to the center of the earth at any arbitrary time; while r^* is for that at $t = 0$. The distance between the flying orbit and the center of the earth is also the radius of the orbit of the satellite being launched. The dimensionless quantity for time is

$$\tau = \sqrt{\frac{r^*}{r}} t, \text{ therefore } d\tau = \sqrt{\frac{r^*}{r}} dt, \text{ or } dt = \sqrt{\frac{r}{r^*}} d\tau. \quad (6.15)$$

The angle θ itself is a dimensionless quantity, consequently, it requires no conversion to dimensionlessness. Since only the circular thrust direction is used, therefore we set $F_r = 0$, while

$$F_\theta = v g^*. \quad (6.16)$$

Here F_θ itself is an acceleration, while the magnitude of v indicates the magnitude of that acceleration. Hence in substituting these dimensionless quantities into Eq. (6.12), we get

$$r^* \frac{r^*}{r^*} \frac{d^2 \rho}{d\tau^2} - r^* \frac{r^*}{r^*} \rho \left(\frac{d\theta}{d\tau} \right)^2 = r^* \frac{1}{\rho^3},$$

or it is the same as

$$\frac{d^2 \rho}{d\tau^2} - \rho \left(\frac{d\theta}{d\tau} \right)^2 = \frac{1}{\rho^3}. \quad (6.17)$$

Substituting these dimensionless quantities into Eq. (6.13), we get

$$\sqrt{\frac{r^*}{r}} r^* \sqrt{\frac{r^*}{r}} \frac{d}{d\tau} \left(\rho^2 \frac{d\theta}{d\tau} \right) = r^* g^* \rho v, \quad (6.18)$$

$$\frac{d}{d\tau} \left(\rho^2 \frac{d\theta}{d\tau} \right) = v \rho.$$

Based on the following three initial conditions, a transformation may be performed on Eq. (6.17); its initial conditions are:

1) When $\tau = 0$,

$$\rho = 1; \quad (6.19)$$

2) When $\tau = 0$, $\rho = 1$, i.e., the velocity of the spaceship in the satellite orbit, which is tangent to the circular orbit or in other

words there is no radial velocity, hence

$$\left(\frac{d\rho}{d\tau}\right)_0 = 0; \quad (6.20)$$

3) When $\tau = 0$, $\rho = 1$, and also its linear velocity or the satellite velocity is $\sqrt{g^* r^*}$, therefore,

$$r^* \left(\frac{d\theta}{dt}\right)_0 = \sqrt{g^* r^*},$$

$$r^* \sqrt{\frac{g^*}{r^*}} \left(\frac{d\theta}{d\tau}\right)_0 = \sqrt{g^* r^*},$$

Hence

$$\left(\frac{d\theta}{d\tau}\right)_0 = 1; \quad (6.21)$$

4) When the initial conditions (1) and (3) are substituted into (6.17), then we obtain

$$\left(\frac{d^2\rho}{d\tau^2}\right)_0 = 0. \quad (6.22)$$

If the results of (6.17) is multiplied by ρ^3 then we get

$$\rho^3 \frac{d^2\rho}{d\tau^2} - \left(\rho^3 \frac{d\theta}{d\tau}\right)^2 = \rho,$$

or

$$\rho^3 \frac{d^2\rho}{d\tau^2} + \rho = \left(\rho^3 \frac{d\theta}{d\tau}\right)^2,$$

therefore

$$\rho^3 \left(\frac{d\theta}{d\tau}\right)^2 = \left[\rho^3 \frac{d^2\rho}{d\tau^2} + \rho\right]^{1/2}.$$

Substituting into Eq. (6.18), we obtain

$$\frac{d}{d\tau} \left(\rho^3 \frac{d^2\rho}{d\tau^2} + \rho \right)^{1/2} = v\rho. \quad (6.23)$$

Equation (6.23) is a nonlinear differential equation of the third order with ρ as the variable and τ as the independent variable. (6.19), (6.20) and (6.22) are the three initial conditions necessary for its solution.

When $\rho(\tau)$ is defined then from the first order integration of (6.18) one can get the circular component velocity $\rho \frac{d\theta}{d\tau}$, while Eq. (6.21) determines the integration constant. This is the principle of seeking a solution. But in order to solve the equation, the relatively more complicated method of numerical integration will have to be used. As to how the numerical integration is carried out, we are not prepared to introduce here.

Following the progress of the flight, when $t = t_1$, the kinetic energy possessed by unit mass of the spaceship is

$$\frac{1}{2} \left[\left(\frac{dr}{dt} \right)_1^2 + \left(r_1 \frac{d\theta}{dt} \right)_1^2 \right],$$

while the corresponding potential energy is

$$-g^* \frac{r^{*2}}{r_1}.$$

The total energy per unit mass is

$$\frac{1}{2} \left[\left(\frac{dr}{dt} \right)_1^2 + \left(r_1 \frac{d\theta}{dt} \right)_1^2 \right] - g^* \frac{r^{*2}}{r_1}.$$

If we desire to conclude the acceleration stage, that is $t = t_1$ then we not only need enough energy (kinetic energy) to overcome the earth's gravitational force in order to leave it but also require to have a definite velocity $n\sqrt{g^* r^*}$; in which n is a dimensionless quantity which indicates the magnitude of the remaining velocity. If we denote the conclusion of the acceleration stage by "1," then

$$\frac{1}{2} \left[\left(\frac{dr}{dt} \right)_1^2 + \left(r \frac{d\theta}{dt} \right)_1^2 \right] - g^* \frac{r^{*2}}{r_1} = \frac{1}{2} (n\sqrt{g^* r^*})^2.$$

After substituting the above dimensionless quantities, the above equation is transformed into

$$\frac{1}{2} \left[r^{*2} \frac{g^*}{r^2} \left(\frac{d\rho}{d\tau} \right)_1^2 + r^{*2} \frac{g^*}{r^2} \rho_1^2 \left(\frac{d\theta}{d\tau} \right)_1^2 \right] - g^* r^* \frac{1}{\rho_1} = \frac{1}{2} n^2 g^* r^*.$$

Namely when $\tau = \tau_1 (t = t_1)$, the dimensionless equation describing the

conclusion of the acceleration stage is

$$\left(\frac{d\rho}{d\tau}\right)_1^2 + \rho_1^2 \left(\frac{d\theta}{d\tau}\right)_1^2 - \frac{2}{\rho_1} = n^2. \quad (6.24)$$

From the differential equation and the boundary conditions, we define first ρ and $\rho \frac{d\theta}{d\tau}$ as functions of τ , then, in actuality, Eq. (6.24) calculates the dimensionless acceleration time τ_1 for every definite interplanetary navigation mission (i.e., fixed n). Having found τ_1 it is possible to determine t_1 , which is the action time of the engine, a very important parameter in the design of spaceships.

Here we shall assume that $v = \text{const}$, that is, thrust varies with the variation of mass but maintains with it a definite ratio. In addition, for the sake of convenience of calculation, we will first assume a larger value of v then from the results to estimate v , i.e., the condition when acceleration is relatively low, since at large v , the acceleration is correspondingly larger. The acceleration stage will not be too long, therefore, it is quite close to 1. Substituting $\rho = 1$ into Eq. (6.23), it is possible then to make use of the initial and final conditions to perform the following approximation integration on Eq. (6.23)

$$\frac{d}{d\tau} \left(\frac{d^2\rho}{d\tau^2} + 1 \right)^{1/2} = v. \quad (6.25)$$

while integrating (6.25) would give us

$$\left(\frac{d^2\rho}{d\tau^2} + 1 \right)^{1/2} = C_1 + v\tau,$$

that is

$$\frac{d^2\rho}{d\tau^2} + 1 = C_1^2 + 2C_1v\tau + v^2\tau^2.$$

Utilizing the initial condition (4), when $\tau = 0$, $d^2\rho/d\tau^2 = 0$, therefore, $C_1 = 1$, then

$$\frac{d^2\rho}{d\tau^2} = 2v\tau + v^2\tau^2. \quad (6.26)$$

Integrate again Eq. (6.26), we obtain,

$$\frac{d\rho}{d\tau} = C_2 + \nu\tau^2 + \frac{\nu^2}{3}\tau^3.$$

Applying initial condition (2), when $\tau = 0$, $d\rho/d\tau = 0$; substituting into the above equation we obtain $C_2 = 0$, then

$$\frac{d\rho}{d\tau} = \nu\tau^2 + \frac{1}{3}\nu^2\tau^3. \quad (6.27)$$

Finally, integrating Eq. (6.27), we obtain

$$\rho = C_3 + \frac{1}{3}\nu\tau^3 + \frac{1}{12}\nu^2\tau^4.$$

Using initial condition (1), we obtain $C_3 = 1$, therefore,

$$\rho = 1 + \frac{1}{3}\nu\tau^3 + \frac{1}{12}\nu^2\tau^4. \quad (6.28)$$

From Eq. (6.28), it can be seen, that when τ is very small, that is in a very short time period, ρ approaches very close to 1, explaining that the above approximation integration is reasonable. Substituting the results of Eq. (6.28) into Eq. (6.18), we obtain

$$\frac{d}{d\tau}\left(\rho^2 \frac{d\theta}{d\tau}\right) = \nu\rho = \nu + \frac{1}{3}\nu^2\tau^3 + \frac{1}{12}\nu^3\tau^4,$$

After integration it becomes,

$$\rho^2 \frac{d\theta}{d\tau} = C_4 + \nu\tau + \frac{1}{12}\nu^2\tau^4 + \frac{1}{60}\nu^3\tau^5.$$

Using the initial condition (3), we obtain $C_4 = 1$, therefore,

$$\rho^2 \frac{d\theta}{d\tau} = 1 + \nu\tau + \frac{1}{12}\nu^2\tau^4 + \frac{1}{60}\nu^3\tau^5. \quad (6.29)$$

Squaring the ρ already found we get

$$\rho^2 = 1 + \frac{2}{3}\nu\tau^3 + \frac{1}{6}\nu^2\tau^4 + \frac{1}{9}\nu^2\tau^6 + \frac{1}{18}\nu^3\tau^7 + \frac{1}{144}\nu^4\tau^8. \quad (6.30)$$

Multiplying the known ρ by $d\rho/d\tau$, we get

$$\rho \frac{d\rho}{d\tau} = \nu\tau^2 + \frac{1}{3}\nu^2\tau^3 + \frac{1}{3}\nu^2\tau^5 + \frac{7}{36}\nu^3\tau^6 + \frac{1}{36}\nu^4\tau^7. \quad (6.31)$$

Multiplying the equation at the conclusion of the acceleration stage,

$t = t_1$, i.e., Eq. (6.24), by ρ_1^2 , we get

$$c \ln \frac{M_0}{M_1} = \sqrt{r^* g^*} \tau_1 v;$$

which is the same as

$$\left(\rho_1 \frac{d\rho}{d\tau}\right)_1 + \left(\rho_1^2 \frac{d\theta}{d\tau}\right)_1 - 2\rho_1 = \rho_1 n^2. \quad (6.32)$$

Substituting the results of (6.29), (6.30) and (6.31) into Eq. (6.32), we obtain

$$\begin{aligned} & \left\{ v\tau_1^2 + \frac{1}{3} v^2 \tau_1^3 + \frac{1}{3} v^2 \tau_1^3 + \frac{7}{36} v^3 \tau_1^4 + \frac{1}{36} v^4 \tau_1^5 \right\}^2 + \left\{ 1 + v\tau_1 + \frac{1}{12} v^2 \tau_1^2 \right. \\ & \quad \left. + \frac{1}{60} v^3 \tau_1^3 \right\}^2 - 2 \left\{ 1 + \frac{1}{3} v\tau_1 + \frac{1}{12} v^2 \tau_1^2 \right\} = n^2 \left\{ 1 + \frac{2}{3} v\tau_1 \right. \\ & \quad \left. + \frac{1}{6} v^2 \tau_1^2 + \frac{1}{9} v^2 \tau_1^2 + \frac{1}{18} v^3 \tau_1^3 + \frac{1}{144} v^4 \tau_1^4 \right\}. \end{aligned} \quad (6.33)$$

Equation (6.33) is the equation obtained by approximation integration after the conclusion of the acceleration stage. But before we proceed to further calculations, let us look at the practical significance of $v\tau_1$, particularly in its relationship with mass ratio. Supposing c is the effective jet velocity; M_0 is the mass at $t = 0$; M_1 is the mass at $t = t_1$; then for unit mass, there is the following relationship on the thrust of unit mass

$$-c \cdot \frac{dM}{dt} \cdot \frac{1}{M} = F_\theta.$$

Since $F_\theta = vg^*$, therefore

$$-c \frac{dM}{M dt} = vg^*.$$

After making it dimensionless

$$-c \frac{dM}{M} = vg^* \sqrt{\frac{r^*}{g^*}} d\tau.$$

Integrating in the interval at τ from 0 to τ_1 and M_0 to M_1 , we get

$$\rho_1^2 \left(\frac{d\rho}{d\tau}\right)_1 + \rho_1^2 \left(\frac{d\theta}{d\tau}\right)_1 - 2\rho_1 = \rho_1 n^2;$$

Hence,

$$\frac{c}{\sqrt{g^* r^*}} \ln \frac{M_0}{M_1} = \tau_1 v. \quad (6.34)$$

Here for a definite propellant, the value of $c/\sqrt{g^* r^*}$ is a constant, while $\sqrt{g^* r^*}$ is actually the orbital velocity of the satellite. In case of the present day chemical rockets, $c/\sqrt{g^* r^*}$ lies between 0.35 to 0.55; for the atomic rockets which we will be discussing in the next chapter $c/\sqrt{g^* r^*}$ is between 1 and 1.25; while for the so-called electric rocket (see the next chapter) $c/\sqrt{g^* r^*}$ can reach 20.

Let $c/\sqrt{g^* r^*} \ln M_0/M_1 = n$, then it actually represent the magnitude of the mass ratio, and may be used as an index of the magnitude of the mass ratio of constant quantities. Therefore $n = v \tau_1$. Substituting this relationship into Eq. (6.33) and we obtain

$$\begin{aligned} & \frac{1}{v^2} \left\{ \eta^2 + \frac{1}{3} \eta^1 + \frac{1}{v^2} \left(\frac{1}{3} \eta^2 + \frac{7}{36} \eta^1 + \frac{1}{36} \eta^0 \right) \right\}^2 + \left\{ (1 + \eta) \right. \\ & \quad \left. + \frac{1}{v^2} \left(\frac{1}{12} \eta^2 + \frac{1}{60} \eta^1 \right) \right\}^2 - 2 \left\{ 1 + \frac{1}{v^2} \left(\frac{1}{3} \eta^2 + \frac{1}{12} \eta^1 \right) \right\} \\ & \quad = n^2 \left\{ 1 + \frac{1}{v^2} \left(\frac{2}{3} \eta^2 + \frac{1}{6} \eta^1 \right) + \frac{1}{v^4} \left(\frac{1}{9} \eta^4 + \frac{1}{18} \eta^3 + \frac{1}{144} \eta^2 \right) \right\}. \end{aligned} \quad (6.35)$$

Using Eq. (6.35) under the condition of given v and, n to find the value of η . Actually, owing to the assumption we introduced at the beginning of our calculation, $\rho \approx 1$. The degree of precision of Eq. (6.35) does not require the use of the factor $1/v^4$, only the use of $1/v^2$ as the factor term is accurate. Hence, consider η as a function of n and v we can expand it into the following form and calculate only up to the factor term $1/v^2$,

$$\eta = \eta^{(0)}(n) + \frac{\eta^{(1)}(n)}{v^2} + \dots \quad (6.36)$$

Combining the $1/(v^2)^0$ terms we get,

$$(1 + \eta^{(0)})^2 - 2 = n^2, \text{ that is, } \eta^{(0)} = \sqrt{2 + n^2} - 1. \quad (6.37)$$

Combining the $1/(v^2)^1$ terms we get

$$\left\{\eta^{(0)s} + \frac{1}{3}\eta^{(0)s}\right\}^2 + 2(1 + \eta^{(0)})\left(\frac{1}{12}\eta^{(0)s} + \frac{1}{60}\eta^{(0)s}\right) + 2(1 + \eta^{(0)})\eta^{(1)} - 2\left\{\frac{1}{3}\eta^{(0)s} + \frac{1}{12}\eta^{(0)s}\right\} = n^2\left\{\frac{2}{3}\eta^{(0)s} + \frac{1}{6}\eta^{(0)s}\right\}.$$

Solving for $\eta^{(1)}$ we get

$$\eta^{(1)} = \frac{\eta^{(0)s}}{2(1 + \eta^{(0)})}\left[\frac{2}{3} - \eta^{(0)} - \frac{13}{15}\eta^{(0)s} - \frac{13}{90}\eta^{(0)s} + n^2\left\{\frac{2}{3} + \frac{1}{6}\eta^{(0)}\right\}\right]. \quad (6.38)$$

First, we use Eq. (6.37) to find $\eta^{(0)}$, then Eq. (6.38) is used to find $\eta^{(1)}$; then from Eq. (6.36) we finally find η . From Eq. (6.38) one can see that: when $n = 0$, it will be sufficient if the energy required for acceleration can overcome the earth's gravitational force. Then, according to Eqs. (6.37) and (6.38) we get

$$n = 0, \begin{cases} \eta^{(0)}(0) = \sqrt{2} - 1 = 0.4142; \\ \eta^{(1)}(0) = + 0.002349. \end{cases}$$

Under the condition when $v > \sqrt{0.10}$, our calculation is sufficiently precise, and also prominently shows: when v varies from a very large value (i.e., $v > \sqrt{0.10}$) to $v = \sqrt{0.10}$, at $n = 0$, the value of η increases from 0.414 to 0.437. When $n = 0$, $v = 0.10$, the above calculation gives $\eta \sim 0.649$ which is not very accurate. When $v < \sqrt{0.10}$ for a definite value of n , the method of numerical integration may be used to find the definite variation relationship between v and the mass ratio. Here we only give the results of our calculations as shown in Fig. 6.4. From the curve, we can see that when acceleration decreases to $1/2$ g, its corresponding mass needs not to be increased too much, which is the same as what we said before that when the acceleration is larger than $1/2$ g, the cost we pay is not excessive, but any continuous decrease below $1/2$ g. The increase in mass ratio will be exceedingly fast. For example: when $n = 0$, $v = 10^{-3}$, the value of η is almost twice the thrust when it is large, which is equivalent to saying that if the velocity of the jet is constant, the logarithm of the mass ratio is twice the large thrust;

but the twofold logarithm is equivalent to saying that at $v = 10^{-3}$ the mass ratio will be the square of the mass ratio at the large thrust. In other words, we have to pay while using small thrust; if we cannot increase the jet velocity then we ought not to make v smaller than 0.10.

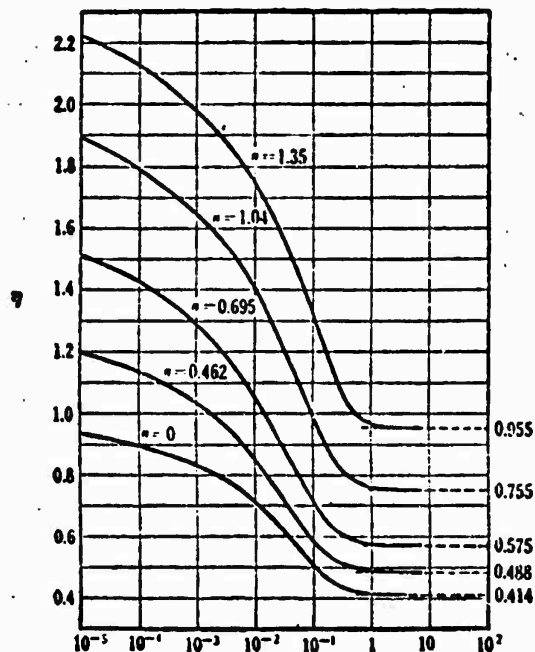


Fig. 6.4. Relationship among η , v , n .

If we can, during the decrease of thrust, simultaneously greatly increase the jet velocity, such as will be described in the next chapter in the case of the engine of electric rockets, e.g., when $v = 10^{-3}$, the value of jet velocity c is greater by better than two times the rocket engine propelled by high thrust chemical propellants or the atomic rocket engines, then the increase in η will be surpassed by the increase in c . The mass ratio will actually decrease, then the condition will be entirely different. The use of low thrust electric rocket engine will have true merit. We will go into this in more detail in the next chapter.

§6.5. LOW THRUST INTERPLANETARY NAVIGATION COURSES

Now we shall return to the real time t , and calculate the time required for the launching from a low earth orbit under low thrust. If $v = 10^{-4}$, $n = 0.462$, i.e., after having left the earth's gravitational field there still remains a velocity of 3.66 km/sec for the entrance into the orbits of other planets, then according to Fig. 6.4 $n = 1.14$, or $\tau_1 = n/v = 1.14/10^{-4} = 1.14 \times 10^4$. But according to Eq. (6.15), $t_1 = \sqrt{\frac{r^*}{g^*}} \tau_1 = \sqrt{\frac{6,371,000}{9.81}} \times 1.14 \times 10^4 = 9.19 \times 10^6 \text{ sec} = 106 \text{ days}$. From this we can see that with the use of such a small thrust, the duration of the acceleration stage has been greatly increased turning it into a portion of the entire interplanetary navigation that can no longer be neglected; while the orbit has also become a very closely twisted spiral as shown in Fig. 6.5.

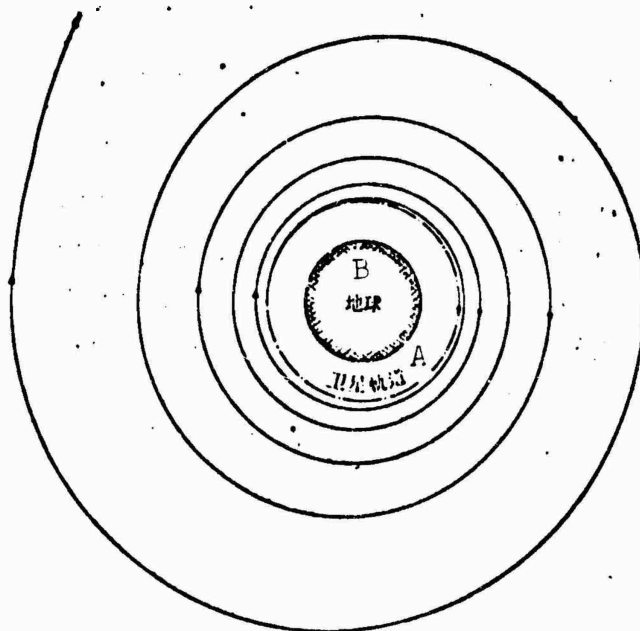


Fig. 6.5. The launching orbit of low thrust rockets. A) Satellite orbit; B) earth.

We can use the results of the above section (§6.4) to calculate the condition of landing on the satellite orbit of a planet at the end of the flight of an interplanetary spaceship. This is merely changing acceleration to deceleration. Figure 6.4 is still applicable, except when the dimensionless quantities are converted to real parameters, the g^* and r^* of the planet must be used.

From the above actual example, we can see that when low thrust is used, it is difficult to distinguish the stages of acceleration when leaving the earth's satellite orbit, the stage of flight between the planets and the stage of approaching the planet. All three stages have become mutually connected. The calculation of the entire orbit is more complex than that for the orbit using large thrust, which itself is also a problem under investigation.

§6.6. LIGHT SAILS

From the facts of small thrust orbits, there have been proposed that the pressure of light may be used to produce a very small acceleration. This will also propel the interplanetary spaceships, or the so-called "light sails" equipped spaceships. Light sails have no expenditure of mass. They rely on the pressure of light to produce thrust. The magnitude of the pressure of light can be explained by a calculation using the conditions in the vicinity of the earth.

Since the intensity of the direct sunlight at the vicinity of the earth is $1.3 \text{ kilowatts/m}^2$, if \dot{m} is the consumption of photon mass per square meter per second, c is the velocity of light ($3 \times 10^{10} \text{ km/sec}$), then

$$1.3 \times 1000 \times 10^7 \text{ erg/m}^2/\text{sec} = \dot{m}c^2,$$

During perpendicular illumination, the light pressure is at a maximum since there are both incident radiation and reflected radiation, thus the light pressure is $2\dot{m}c$, hence

$$2\pi\dot{r}c = \frac{2 \times 1.3 \times 10^{10}}{3 \times 10^{10}} = 0.866 \text{ dyne/m}^2$$

that is, 1000 m² sails can produce thrust of 1/1000 kilograms. If the weight of the flying vehicle is 100 kg, then its velocity will be 1/100,000 g corresponding to approximately $v = 10^{-5}$. It has been carefully computed that sail ships such as that, if it is propelled out of the earth's gravitational field by a rocket of large thrust, then it will take approximately a year to reach the area around Mars. Hence, if we do not consider the problems in construction, and look at it purely from an orbit standpoint, light sails are feasible. But, this is a rather theoretical way of thinking, there are quite a few things that require to be studied, e.g., the sails have to be made very thin and light and then its strength will be very low easily broken by the shooting stars and meteors in space. Consequently, the realization of the use of light sails still awaits further research.

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[Footnotes]

- 15 What we can point out here is: in the calculation of dynamics, for simplification of calculation, it is very common that the method of changing the variables into dimensionless quantities is used. This will not only simplify the form of calculation but at the same time since the variables are dimensionless, during calculation, it is not only convenient but will not get into errors due to nonunified dimensions. Therefore, the method of dimensionless variables is a most frequently used practical technique. As to how to select the basic measuring units, it is necessary to have a concrete analysis of the problem on hand and make it so that after changing the variables into dimensionless quantities, the calculation will become very much simpler.